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The Resonant Dynamic Magnetization Distribution in Ferromagnetic Thin Film with the Antidot

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The influence of homogeneous dynamic microwave magnetic field applied to the permalloy (Py) thin film with a single circular antidot on the magnetization dynamics was theoretically investigated. It was considered that the Py film is saturated by the external constant magnetic field along the direction perpendicular to the film plane. The linearized Landau-Lifshitz equation was applied in order to create an analytical model of small deviations from the equilibrium values of the magnetization and magnetic field. Conditions of the local ferromagnetic resonances were defined and the dependency of resonance frequency on the magnetic field magnitude was visualized. The model has shown that the amplitude of the resonant magnetization oscillations is localized near the antidot edge and their position is dependent on the frequency.

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1. Introduction

Spin waves (SWs) are investigated intensively during last decades, both theoretically and experimentally. Numerous papers are dedicated to SWs in various types of media, including spread of SWs in thin ferromagnetic films [1, 2].

In this paper, the spatially dependent ferromagnetic resonance in ferromagnetic thin film with one antidot under influence of the external magnetic field perpendicular to the sample surface was investigated. In order to create an analytical model of small deviations from the equilibrium values of the magnetic moment and magnetic field, the linearized Landau-Lifshitz equation [3] was solved as an eigen-problem in the direct space. It was found that the main reason of non-uniform oscillations occurrence is a magnetostatic field [4, 5] caused by the presence of the antidot. The model has shown that there is a maximum of the demagnetization field amplitude localized near the antidot edge and the amplitude decreases with the distance from the edge increasing. Conditions of the local ferromagnetic resonances which are different at different distances from the edge because of non-uniformity of the magnetostatic field were also defined.

2. Analytical model

The influence of dynamic microwave magnetic field applied to the Py thin film of thickness h with a single circular antidot on the magnetization was theoretically investigated. We assume the Py film is saturated by the external constant magnetic field along the direction perpendicular to the film plane, the z axis (Fig.1). Coordinates x and y define the film plane. To calculate the magnetic field distribution in the film and to create an analytical model of small eigen-oscillations from the equilibrium values of the magnetization and magnetic field we solve the Landau-Lifshitz equation which describes magnetization dynamics and is the following:

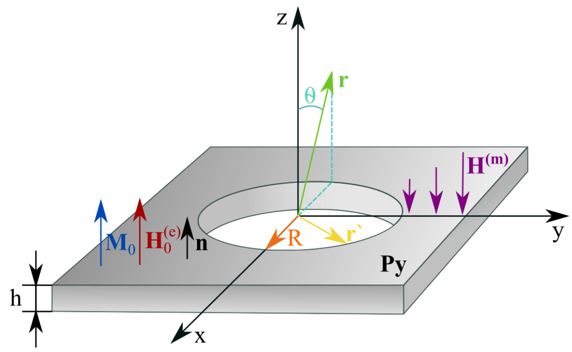

Fig. 1. Direction of the external magnetic field $\boldsymbol{H}_0^{(e)}$, demagnetization field $\boldsymbol{H}^(m)$ and magnetization $\boldsymbol{M}_0$ in the Py film of thickness h with one antidot of radius R.

$$\frac{\partial \boldsymbol{M}}{\partial t} = g(\boldsymbol{M} \times \boldsymbol{H}), \tag{1}$$

where $\boldsymbol{M}$ is the magnetization vector, g — gyromagnetic ratio, and $\boldsymbol{H}$ is the effective magnetic field:

$$\boldsymbol{H}(\boldsymbol{r}, t) = -\frac{\delta W}{\delta \boldsymbol{M}(\boldsymbol{r}, t)}. \tag{2}$$

The macroscopic energy of ferromagnetic material can be written as:

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$$W = \int\limits_V \mathrm{d}\boldsymbol{r}\left\{\frac{1}{2}\alpha\left(\frac{\partial \boldsymbol{M}}{\partial x_i}\right)^2 - \frac{1}{2}\boldsymbol{M}\boldsymbol{H}^{(m)} - \boldsymbol{M}\boldsymbol{H}_0^{(e)} + \frac{1}{2}\beta\left(\boldsymbol{M}\boldsymbol{n}\right)^2\right\}, \quad (3)$$

where $\frac{1}{2}\alpha\left(\frac{\partial \boldsymbol{M}}{\partial x_i}\right)^2$ is density of the exchange energy, $\alpha = 2A/M_0^2$ with A exchange constant and M_0 saturation magnetization, $\boldsymbol{M}\boldsymbol{H}^{(m)}/2$ is the density of magnetostatic energy, $\boldsymbol{M}\boldsymbol{H}_0^{(e)}$ — density of the Zeeman energy, $\frac{1}{2}\beta\left(\boldsymbol{M}\boldsymbol{n}\right)^2$ — density of the magnetic anisotropy energy.

The magnetostatic field $\boldsymbol{H}^{(m)}$ can be calculated using the following magnetostatic equations:

$$\mathrm{rot}\,\boldsymbol{H}^{(m)} = 0, \quad \mathrm{div}\left(\boldsymbol{H}^{(m)} + 4\pi\boldsymbol{M}\right) = 0. \quad (4)$$

The solution of the first equation can be written as:

$$\boldsymbol{H}^{(m)}\left(\boldsymbol{r}\right) = -\mathrm{grad}\,\psi\left(\boldsymbol{r}\right), \quad (5)$$

where $\psi\left(\boldsymbol{r}\right)$ — magnetostatic potential. Then from the second equation we get [1]:

$$\psi\left(\boldsymbol{r}\right) = \int \mathrm{d}\boldsymbol{r}' M_i\left(\boldsymbol{r}'\right)\frac{\partial}{\partial x_i'}\frac{1}{\left|\boldsymbol{r}-\boldsymbol{r}'\right|}, \quad (6)$$

$$\frac{1}{\left|\boldsymbol{r}-\boldsymbol{r}'\right|} = \frac{1}{\sqrt{\left(x-x'\right)^2 + \left(y-y'\right)^2 + \left(z-z'\right)^2}}. \quad (7)$$

Following the approach presented in [3]: we solve Eq. (6) by defining the dot, instead of the film with antidot, with the auxiliary magnetization anti-parallel to the z-axis:

$$\boldsymbol{M_i} = \left(0, 0, -M_0\right). \quad (8)$$

Taking Eq. (8) into account, and substituting Eq. (6) into Eq. (5) the solution can be written as follows:

$$\boldsymbol{H}^{(m)}\left(\boldsymbol{r}\right) = M_0\mathrm{grad}\left(\int_0^h \mathrm{d}z'\int_{-\sqrt{R^2-x'^2}}^{\sqrt{R^2-x'^2}}\mathrm{d}y'\int_{-R}^{R}\mathrm{d}x' \times\frac{\partial}{\partial z'}\frac{1}{\left|\boldsymbol{r}-\boldsymbol{r}'\right|}\right). \quad (9)$$

In Eq. (9) the integration is made over the dot volume.

Transformation of Eq. (9) to dimensionless units (units of dot radius) and then to cylindrical coordinates allows to decompose integrand over the Legendre polynomials using following expression:

$$\int\limits_0^R\int\limits_0^{2\pi}\frac{1}{\left|\boldsymbol{r}-\boldsymbol{r}'\right|}r'\,\mathrm{d}r'\,\mathrm{d}\alpha' = 2\pi\sum_{n=0}^{\infty}\frac{\left(-1\right)^n\left(2n\right)!}{2^{2n}\left(n!\right)^2}P_{2n}\left(\cos\theta\right)\frac{R^{2n+2}}{\left(2n+2\right)r^{2n+1}}, \quad (10)$$

where $P_{2n}\left(\cos\theta\right)$ is Legendre polynomial. First four terms of the polynomial expansion are taken into account in the following, and integrate Eq. (9) with the expressions:

$$\frac{h}{R} \ll 1, \quad \cos\theta = \frac{z}{r}, \quad r = \sqrt{x^2+y^2+z^2}. \quad (11)$$

Substituting Eq. (10) into Eq. (9) and taking field from the middle plane of the dot ($z = 0$), the magnetostatic field induced by the antidot in the ferromagnet can be written as $\boldsymbol{H}^{(m)} = \left(H_x^{(m)}, H_y^{(m)}, H_z^{(m)}\right)$, where the x, y-components of the field are zero, and the z-component of the field is:

$$H_z^{(m)} = 2\pi h M_0\frac{\partial^2}{\partial z^2}\sum_{n=0}^{\infty}\frac{\left(-1\right)^n\left(2n\right)!}{2^{2n}\left(n!\right)^2}P_{2n}\left(\cos\theta\right) \times\frac{R^{2n+2}}{\left(2n+2\right)r^{2n+1}}, \quad (12)$$

The magnetostatic field from Eq. (12) needs to be supplemented with the demagnetizing field induced by the film equal $-4\pi M_0$, which is taken into account in the following part.

After linearization, the Landau-Lifshitz equation (1) reads [3]:

$$\frac{\partial \boldsymbol{m}}{\partial t} = g\boldsymbol{M}_0\times\left[\boldsymbol{h}^{(e)} + \alpha_{ik}\frac{\partial^2\boldsymbol{m}}{\partial x_i\partial x_k} + \beta\left(\boldsymbol{m}\boldsymbol{n}\right)\boldsymbol{n} - \frac{1}{M_0^2}\left(\boldsymbol{M}_0\boldsymbol{H}_0^{(i)} + \beta\left(\boldsymbol{M}_0\boldsymbol{n}\right)^2\right)\boldsymbol{m}\right], \quad (13)$$

where $\boldsymbol{m} = \left(m_x, m_y, 0\right)$ is the vector of dynamical component of the magnetization, $\boldsymbol{M}_0 = \left(0, 0, M_{0z}\right)$, $\boldsymbol{H}_0^{(i)} = \boldsymbol{H}_0^{(e)} + \boldsymbol{H}^{(m)}$, $\boldsymbol{H}_0^{(e)} = \left(0, 0, H_0^{(e)}\right)$. We have included in Eq. (13) the external microwave magnetic field $\boldsymbol{h}^{(e)} = \left(h_0^{(e)}, 0, 0\right)$ which is homogenous in space.

Eq. (13) transforms to the set of equations for the dynamical magnetization components:

$$\begin{cases}\dfrac{\partial m_x}{\partial t} = -gM_0\left(\alpha\Delta m_y - f_0\left(x,y\right)m_y\right)\\ \dfrac{\partial m_y}{\partial t} = gM_0\left(h_0^{(e)} + \alpha\Delta m_x - f_0\left(x,y\right)m_x\right)\\ \dfrac{\partial m_z}{\partial t} = 0\end{cases}, \quad (14)$$

with $f_0\left(x,y\right) = \frac{H_0^{(e)}}{M_0} + \frac{H_z^{(m)}}{M_0} + \beta$. In the following the exchange terms, i.e. terms with Δm_x and Δm_y in Eq. (14) are neglected, which is justified when the condition $\alpha\left(\left(2R\right)^2 f_0\left(x,y\right)\right)^{-1} \ll 1$ is fulfilled and the condition required by linear approximation $m \ll M_0$ is valid.

Under these approximations, the solutions of Eq. (14) have the form of the monochromatic oscillations:

$$m_x = m_x\left(x,y\right)\sin\omega t, \quad m_y = m_y\left(x,y\right)\cos\omega t, \quad (15)$$

if the external microwave magnetic field is:

$$h_0^{(e)}\left(t\right) = h_0^{(e)}\sin\omega t, \quad (16)$$

where $\omega$is the angular frequency.

Substituting Eqs. (15) and (16) into Eq. (14) the following is obtained:

$$\begin{cases}\dfrac{m_x}{h_0^{(e)}} = \dfrac{g^2M_0\left(H_0^{(e)} + H_z^{(m)}\right)}{g^2\left(H_0^{(e)} + H_z^{(m)}\right)^2 - \omega^2}\\ \dfrac{m_y}{h_0^{(e)}} = \dfrac{g\omega M_0}{g^2\left(H_0^{(e)} + H_z^{(m)}\right)^2 - \omega^2}\end{cases}, \quad (17)$$

which is the solution of Eq. (13).

Ferromagnetic resonance (FMR) frequencies can be obtained from the fraction denominator in Eq. (17) equated to zero:

$$\omega = g\left(H_0^{(e)} + H_z^{(m)}\right). \tag{18}$$

3. Results

In the calculations we consider the following parameters: external magnetic field $\boldsymbol{H}_0^{(e)} = 15$ kOe, magnetization saturation $\boldsymbol{M}_0 = 860$ G, radius of the antidot $R = 10^{-5}$ cm, film thickness $h = 10^{-6}$ cm, and the exchange constant $A = 1.3 \times 10^{-6}$ erg/cm.

Eq. (12) visualized in Fig. 2(a) shows that there the static component of the magnetostatic field has negative sign and the amplitude increases with increasing distance from the edge up to the value $-4\pi M_0$. The condition of the local ferromagnetic resonances defined in Eq. (18) shows similar character in the area around the antidot (Fig. 2b).

In view of cylindrical symmetry of the problem the dependence of dynamical magnetization amplitude in resonance on the distance from the edge of antidot is shown in Fig. 2c. The spectra are plotted for a selected excitation frequencies f ($\omega = 2\pi f$) from the range shown in Figs. 2a–b. The amplitude of magnetization oscillations is localized at the narrow ring around antidot. The position of the amplitude maximum shifts farther from the antidot edge with decreasing frequency from 15 GHz to 12 GHz, accordingly with the resonance frequencies shown in Fig. 2b. This change is complemented with the increase of the maximum amplitude. The increase of the amplitude happens up to the FMR frequency of the plane ferromagnetic film (11.446 GHz).

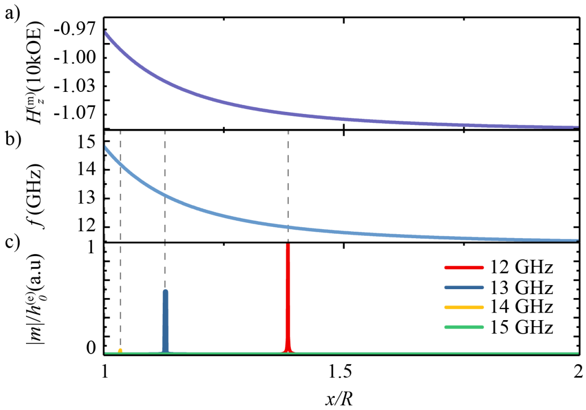

Fig. 2. a) Distribution of the z-component of the magnetostatic field, b) distribution of the resonance frequency of the magnetization oscillations in a thin Py film with an antidot. c) Dependency of the absolute value of the dynamic magnetization perturbation over excitation with homogeneous small deviation of the magnetostatic field in a whole thin film for selected frequencies from the range shown in Fig. 2b.

Results from analytical model were successfully confirmed using micromagnetic simulations with MuMax3 [6].

4. Conclusions

We developed the analytical model allowing for the calculation of the spatially dependent FMR frequency in a perpendicularly magnetized thin film with the antidot, taking into account inhomogeneity of the demagnetizing field. The model showed that maximum amplitude of the static component of the magnetostatic field is localized near the antidot edge and the amplitude decreases with increasing the distance from the edge. The conditions of the local FMRs were defined and the spatial distribution of the resonance frequency of oscillations have been calculated. Similar effects shall exist also for other shapes of antidots, however the in-plane distribution will be anisotropic.

In view of cylindrical symmetry near the antidot, a change of the dynamic x- and y-components of the magnetization in resonance have the same symmetry. The localization and amplitude of the dynamic component of magnetization at resonance depends on frequency. The highest amplitude is located in further distances from the edge at frequencies closer to the FMR frequency of the plane ferromagnetic film. With increasing frequency, the amplitude concentration shifts towards the antidot edge.

Acknowledgments

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